Simulation of Multiple Coulomb Scattering and Comparison with Recent Data

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OUTLINE

- Introduction
- Scattering off Atomic Electrons and Moliere Theory
- Monte Carlo Algorithms
- Comparison with MuScat data
- Mean-squared angle
- Energy-angle correlations
- Effect on muon cooling simulations

INTRODUCTION

XX century:

- Moliere-Fano theory of small angle multiple Coulomb scattering.
- B. Gottschalk et al performed comparison of seven experiments on multiple Coulomb scattering and Moliere theory with Fano correction. These measurements range from 1 MeV to 200 GeV incident energy from beryllium to uranium. The distribution of deviation from theory for 39 independent measurements is approximately normal, with mean value of $-0.3 \pm 0.5\%$ and *rms* spread 3%.
- Lynch&Dahl pointed out that measured value for the width of multiple scattering distribution on hydrogen at 50 - 200 GeV/c disagrees with Moliere-Fano theory more than 14 standard deviations! Bethe $Z(Z+1)$ model agrees within error with this data.
- Best Monte-Carlo codes based on $Z(Z+1)$ extension of Moliere theory.

INTRODUCTION (2)

XXI century:

- New modifications to the theory: Tollestrup & Monroe, Striganov
- MuScat experiment
- New Monte Carlo codes: ELMS, GEANT4, SAMCS

Let's look at new developments and old problem.

MOLIERE THEORY

An angular distribution of a charged particle after passing through an absorber of a thickness t, can be written as

$$F(\theta,t) = \frac{1}{2\pi} \int_0^{\infty} J_0(p\theta) exp(-tA(p)) p dp, \quad (1)$$

where

$$tA(p) = t \int_0^{\infty} d\Omega (1 - J_0(p\theta)) \frac{d\Sigma}{d\Omega} \quad (2)$$

Elastic scattering cross section reads

$$t \frac{d\Sigma_{el}}{d\Omega} = \frac{\chi_c^2 q_{el}(\theta,\chi_a)}{\pi\theta^4}$$

Final answer is can be written as

$$F(\theta,t) = \frac{1}{\pi\theta_M^2} (exp(-x) + f^{(1)}(x)/B + f^{(2)}(x)/B^2),$$

where

$$B - lnB = ln(\chi_c^2/\chi_a^2) + 1 - 2C$$

$$x = \theta^2/\theta_M^2 \text{ and } \theta_M^2 = B\chi_c^2.$$

SCATTERING OFF ATOMIC ELECTRONS (1)

A charged particle traversing medium is deflected primarily by elastic collisions in the Coulomb field of nuclei. Inelastic collisions with atomic electrons also should be taken into account. To estimate a contribution of inelastic collisions, Bethe proposed to replace the squared nuclear charge Z^2 with the sum of the squares of the nuclear and electronic charges $Z(Z+1)$. This procedure would be accurate if the single scattering cross sections were the same for nucleus and electron targets. The actual cross sections are different at small and large angles.

Fano modified Moliere theory taking into account above differences. He proposed solutions for electron and heavy particles. Recently, Tollestrup and Monroe independently reconsidered Moliere theory for low-Z materials. They obtained formulae similar to Fano's electron and heavy particle distributions, but in Tollerstrup&Monroe approach this results have different physical meaning.

SCATTERING OFF ATOMIC ELECTRONS (2)

Tollerstrup&Monroe proposed some approximative model. Recently we modified Moliere-Fano theory using simple single scattering inelastic cross section but without any other approximations.

The recoil imparted to atomic electron by incident heavy particle cannot exceed a certain limit, so a simple approximation of the inelastic cross section is given by

$$t\frac{d\Sigma_{in}}{d\Omega} = \frac{\chi_c^2 q_{in}(\chi)}{Z\pi\chi^4}, \chi \leq \chi_{max}$$

χ_{max} can be calculated using energy and momentum conservation laws. We prefer to define χ_{max} so, that the mean-squared angle resulting from this simple formula is adjusted to the mean-squared angle calculated from the precise cross section.

SCATTERING OFF ATOMIC ELECTRONS (3)

Angular distribution can be written as

$$F(\theta,t) = \frac{1}{2\pi}\int_0^{\infty} du u J_0(\frac{\theta u}{\theta_M}) exp(-u^2/4 + A(u,B))$$

$$A(u,B) = u^2 log(u^2/4)/4B) + I_{in}$$

$$I_{in} = 2u^2/(Z+1)/B \int_{u\chi_{max}/\theta_M}^{\infty} dx(1-J_0(x))/x^3$$

and B is defined from

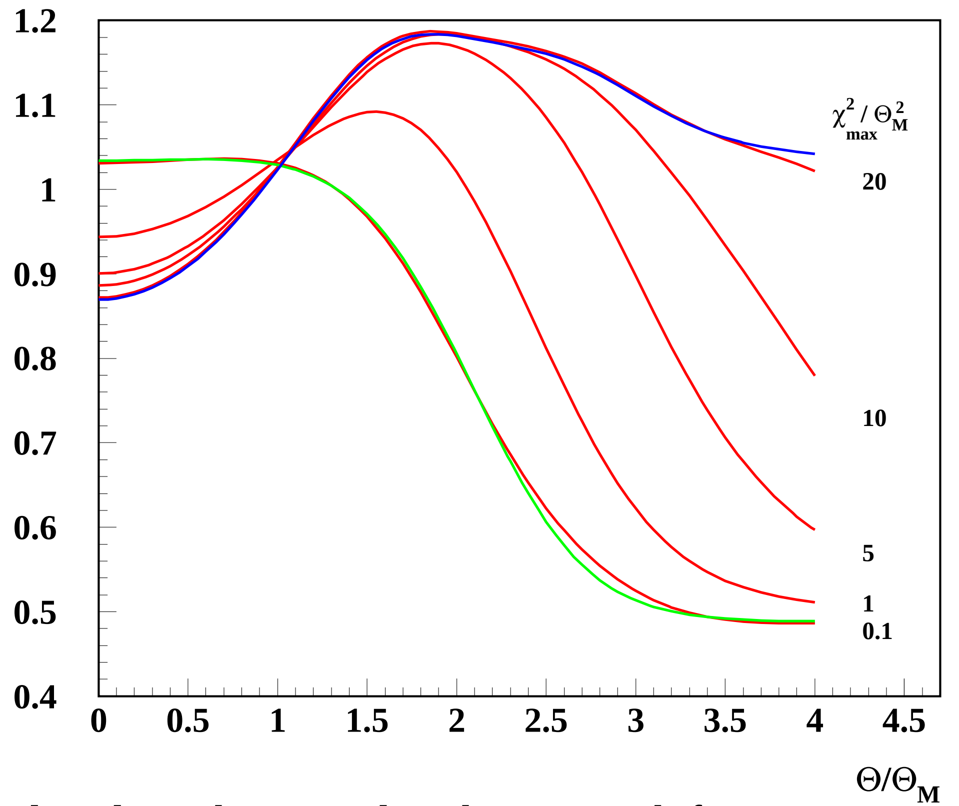
$$B - lnB = ln(\chi_c^2/\chi_a^2) + 1 - 2C + ln(\chi_a^2/\chi_i^2)/(Z+1)$$

For thin target ($w = \chi_{max}/\theta_M \gg 1$), correction term $I_{in} = 0$ and we got Moliere distribution with redefined B -Fano's electron solution

For thick target ($w = \chi_{max}/\theta_M \ll 1$), we also got Moliere distribution, but B should be defined from (Fano's solution for heavy particles)

$$B - lnB = ln(\chi_c^2/\chi_a^2) + 1 - 2C + ln(\chi_{max}^2/\chi_i^2)/Z$$

PROPOSED THEORY *vs* BETHE'S Z(Z+1)



Ratio of the modified Moliere theory and Bethe approach for 200 MeV/c muon on a hydrogen absorber. Red lines is a new solution, green line is low *w* limit, blue line is high *w* limit.

THIN AND THICK TARGETS

The low (thick target) and high (thin target) w limits have been obtained by Fano. He believed that high-w solution is valid for incident electrons and low-w solution can be applied for heavy particles. Our consideration shows that the above limits have different ranges of applicability. If $w \gg 1$, high-w solution should be used even for heavy particles. As shown in Table 1, this conclusion is supported by experiment.

Table 1 : Ratio of calculated and measured widths of angular distributions.

element	β	w	Bethe/Exp	Fano(heavy)/Exp	Fano(elect)/Exp
H	0.14	0.058	1.12 ± 0.02	1.03 ± 0.02	1.19 ± 0.02
H	0.17	0.134	1.03 ± 0.03	0.99 ± 0.03	1.11 ± 0.03
H	0.17	0.078	1.09 ± 0.02	1.01 ± 0.02	1.16 ± 0.02
H	1.00	15	1.01 ± 0.01	1.14 ± 0.01	1.04 ± 0.01
D	0.14	0.058	1.11 ± 0.03	1.02 ± 0.03	1.18 ± 0.02
He	0.14	0.069	1.09 ± 0.02	1.03 ± 0.02	1.12 ± 0.02

MONTE CARLO MODELS (1)

The simulation algorithms can be classified into three different classes:

- “detailed” - all collisions are simulated
- “condensed” - global effects of collisions is simulated after track segment using theory or its approximation
- “mixed” - “hard” collisions are simulated one by one, effect of “soft” collisions added after given step

Current Monte Carlo codes:

- GEANT4 - “condensed” algorithm based on approximation of Lewis-Moliere theory
- ELMS - “mixed” algorithm based on cross section derived from first principles, full data base is available for hydrogen only (?)
- GEANT3, ICOOL(before April 2006) - “condensed” algorithm based on Bethe $Z(Z+1)$ theory
- SAMCS - “mixed” algorithm implemented in MARS15

SAMCS - MONTE CARLO MODELS (2)

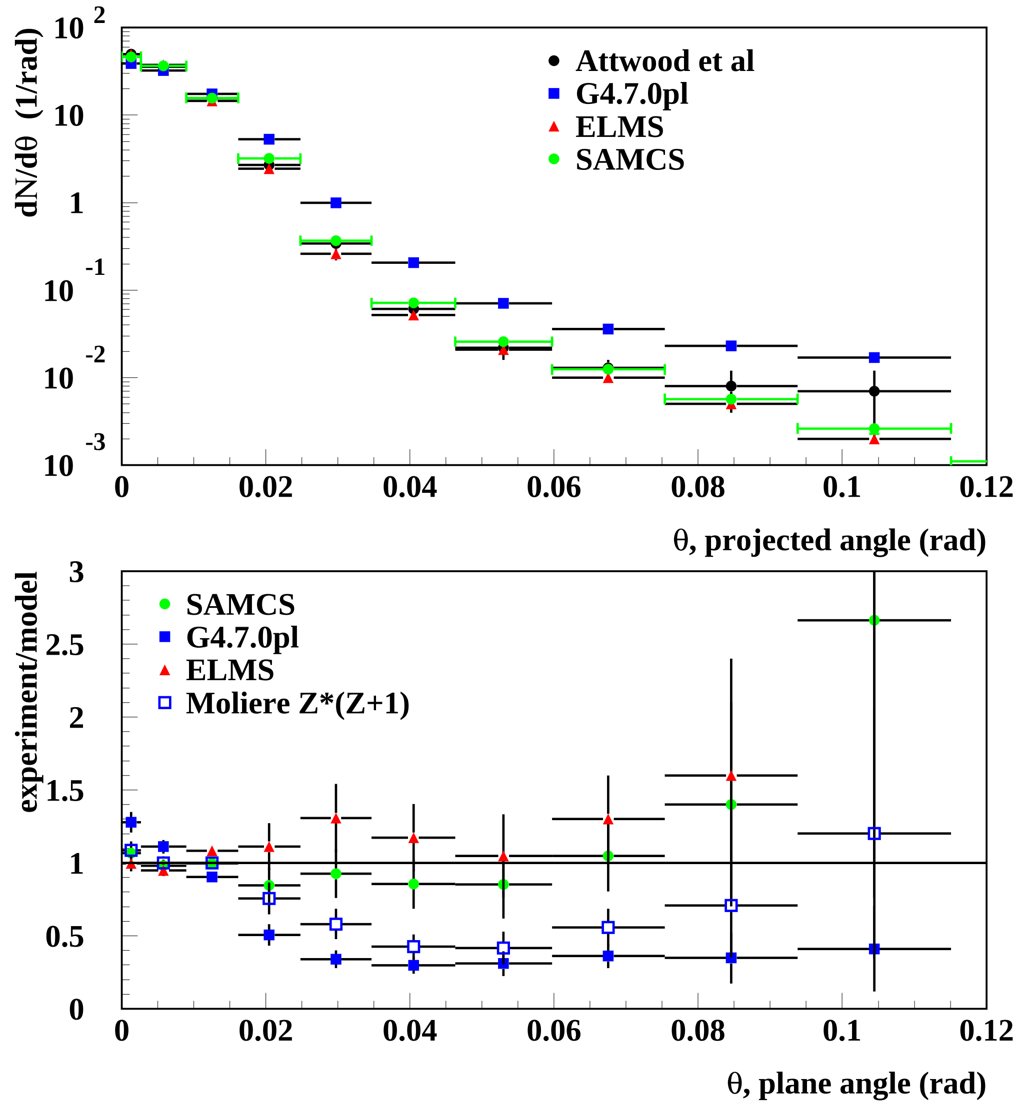
An efficient method to simulate multiple scattering is based on a separate treatment of "soft" and "hard" interactions. Angular deflection in a large number of "soft" collisions is sampled from a "continues" distribution, "hard" scattering are simulated explicitly. There is an obvious correlation between precision and efficiency of the algorithm and the value of a boundary angle θ_b between "soft" and "hard" collisions. For small θ_b, a number of discrete interactions is large and precision is high, for larger θ_b, the efficiency increases but the accuracy decreases.

In SAMCS "continues" angular distribution is given by

$$F_c(\theta,t) = \frac{1}{\pi < \theta_s^2 >} e^{-\phi_s} (1 + r_s L_2(\phi_s) + 3 r_s^2 L_4(\phi_s) + \ldots),$$

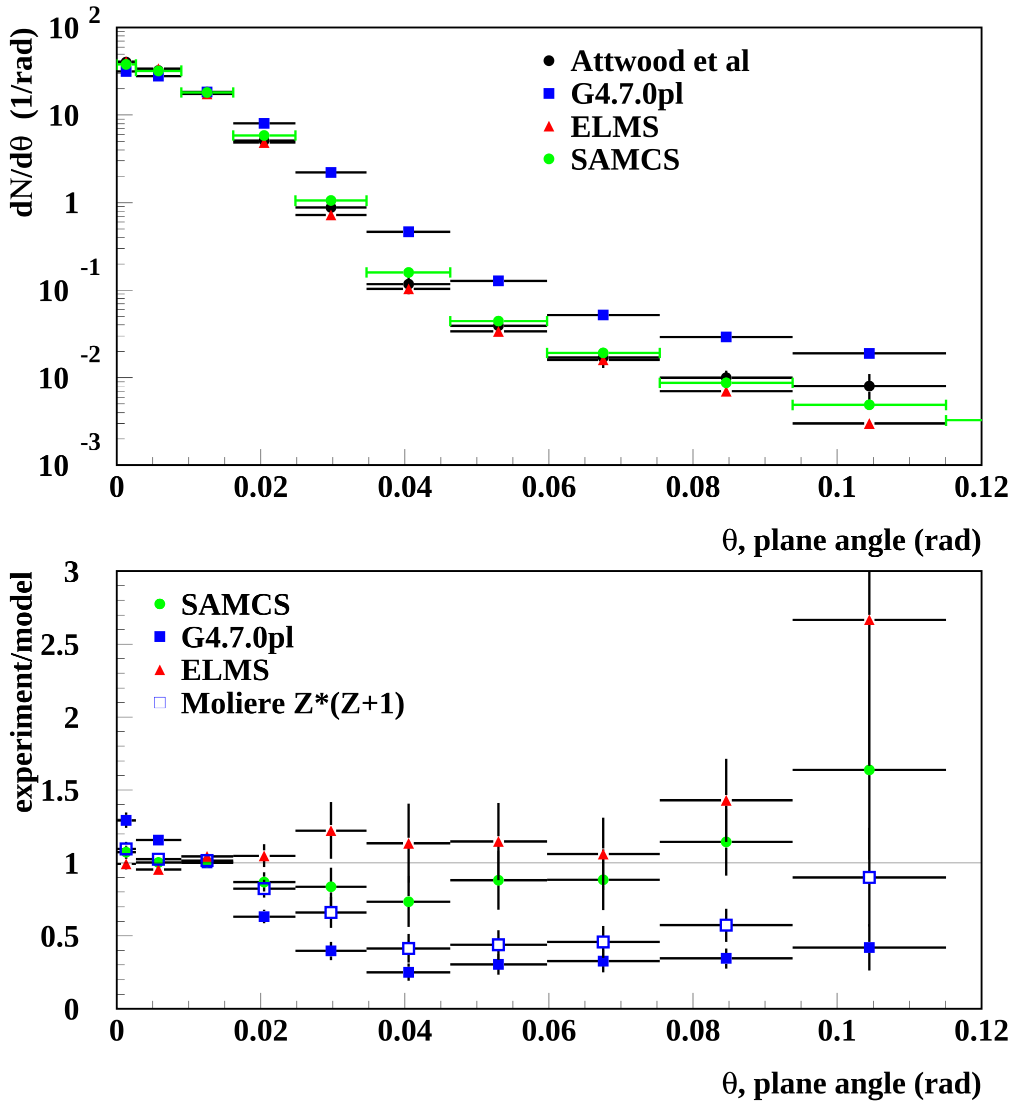
where $\phi_s = \frac{\theta^2}{<\theta_s^2>}$, $r_s = \frac{<\theta_s^4>}{2<\theta_s^2>^2}$, $< \theta^k >= t \int_0^{\theta_b} d\Omega \theta^k \frac{d\Sigma}{d\Omega}$, L_k are Laguerre polynomials.
"Continues" energy loss distribution are described by modified Vavilov distribution.
Simulation of "hard" collisions takes into account projectile and nucleus charged distributions and exact kinematics of a projectile-electron interactions.

MODELS *vs* MuScat MEASUREMENTS



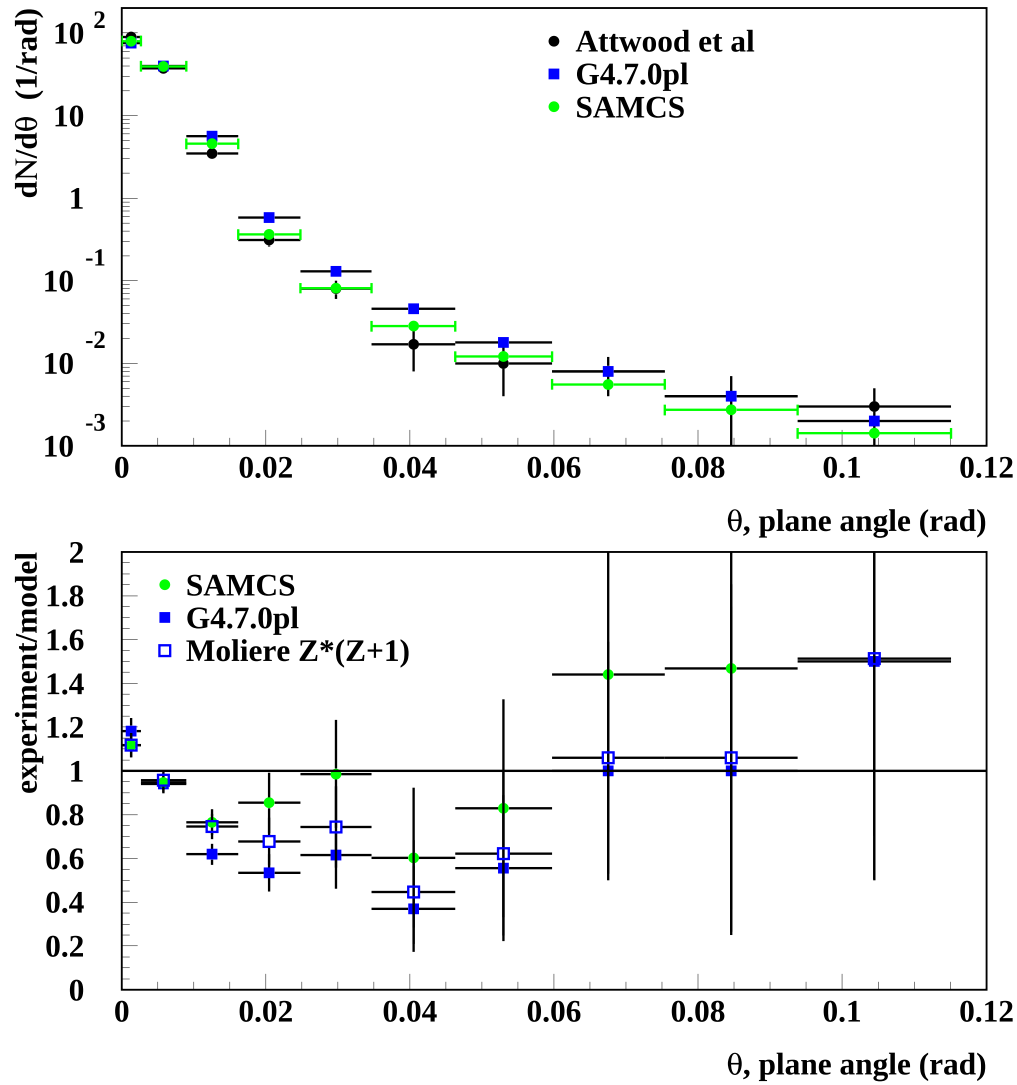
Angular distribution of 172 MeV/c muon after 109 mm of liquid hydrogen

MODELS *vs* MuScat MEASUREMENTS



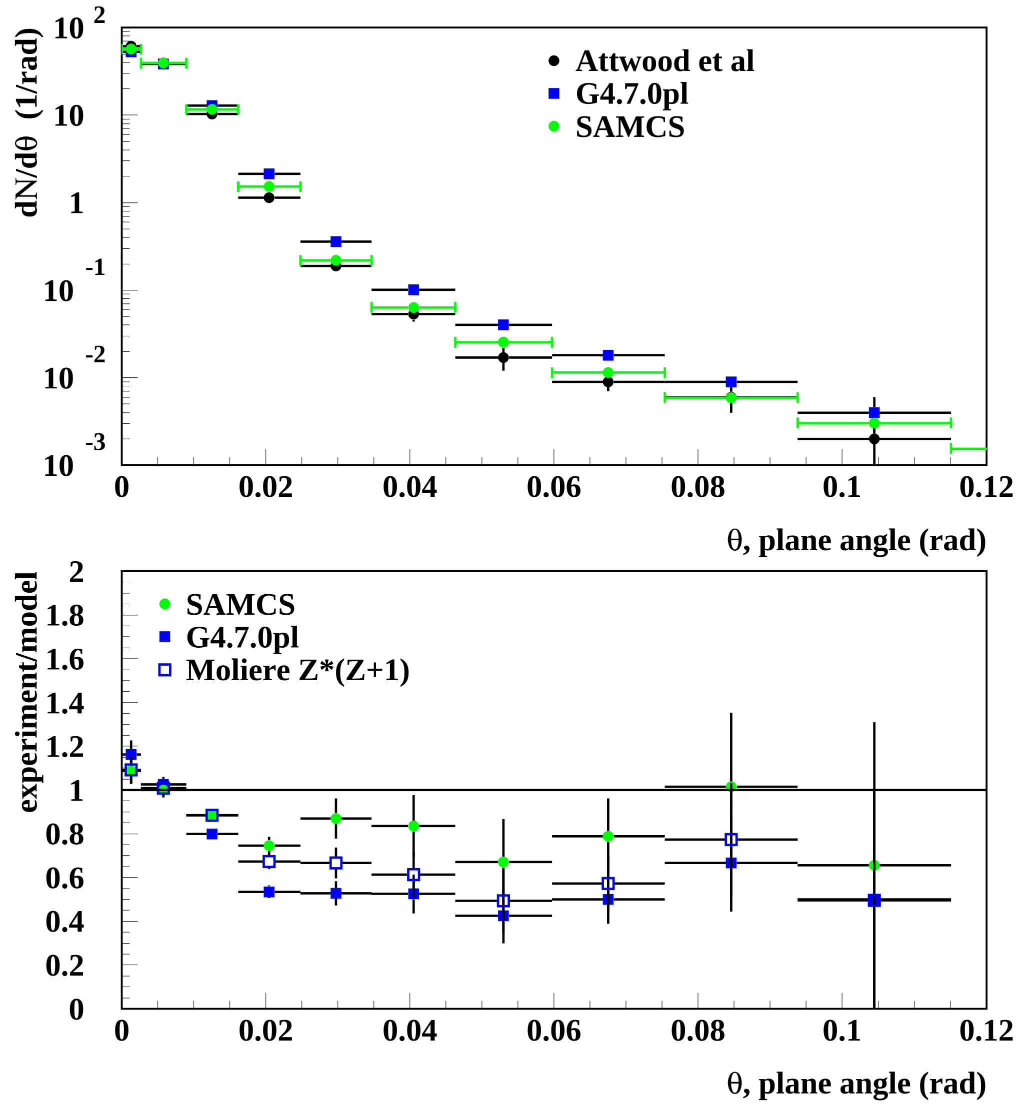
Angular distribution of 172 MeV/c muon after 159 mm of liquid hydrogen

MODELS *vs* MuScat MEASUREMENTS



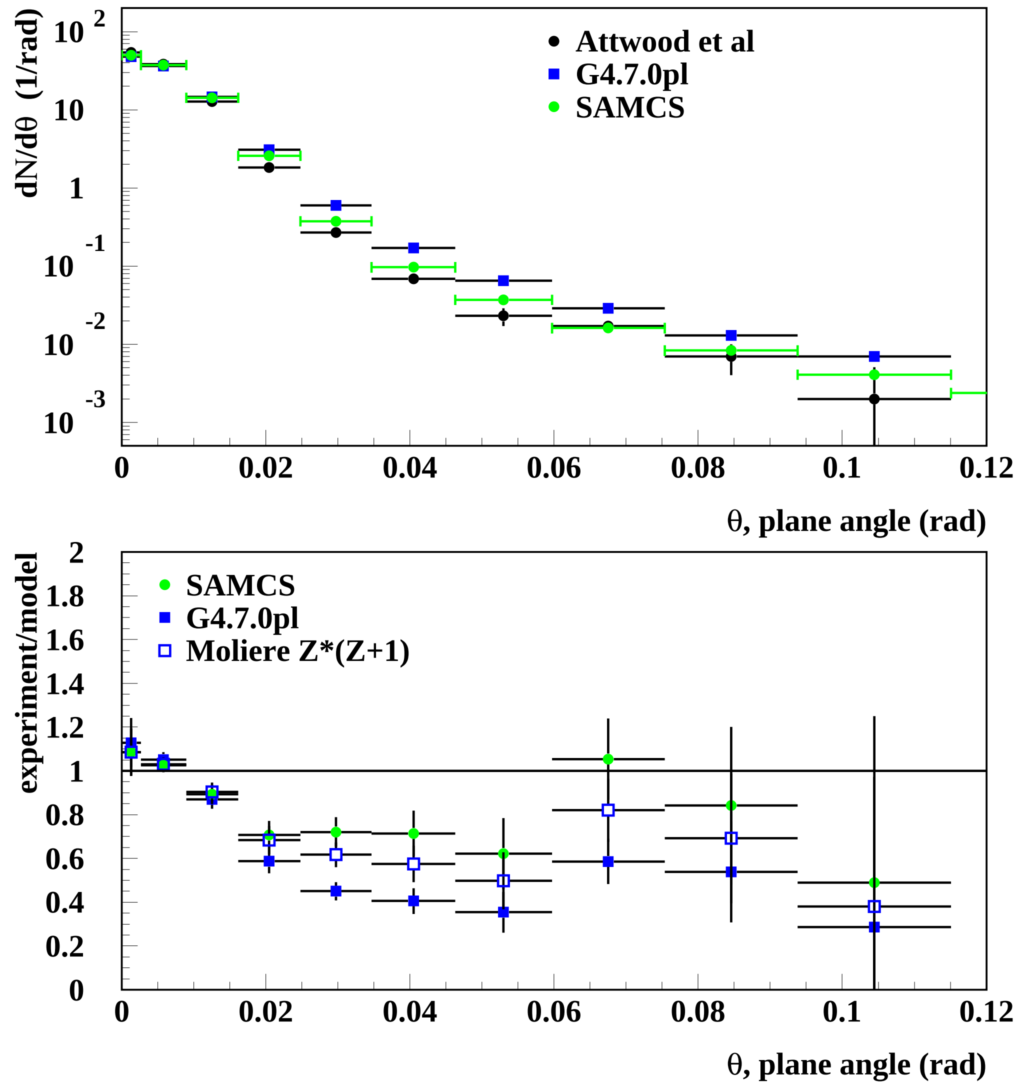
Angular distribution of 172 MeV/c muon after 6.415 mm of lithium

MODELS *vs* MuScat MEASUREMENTS



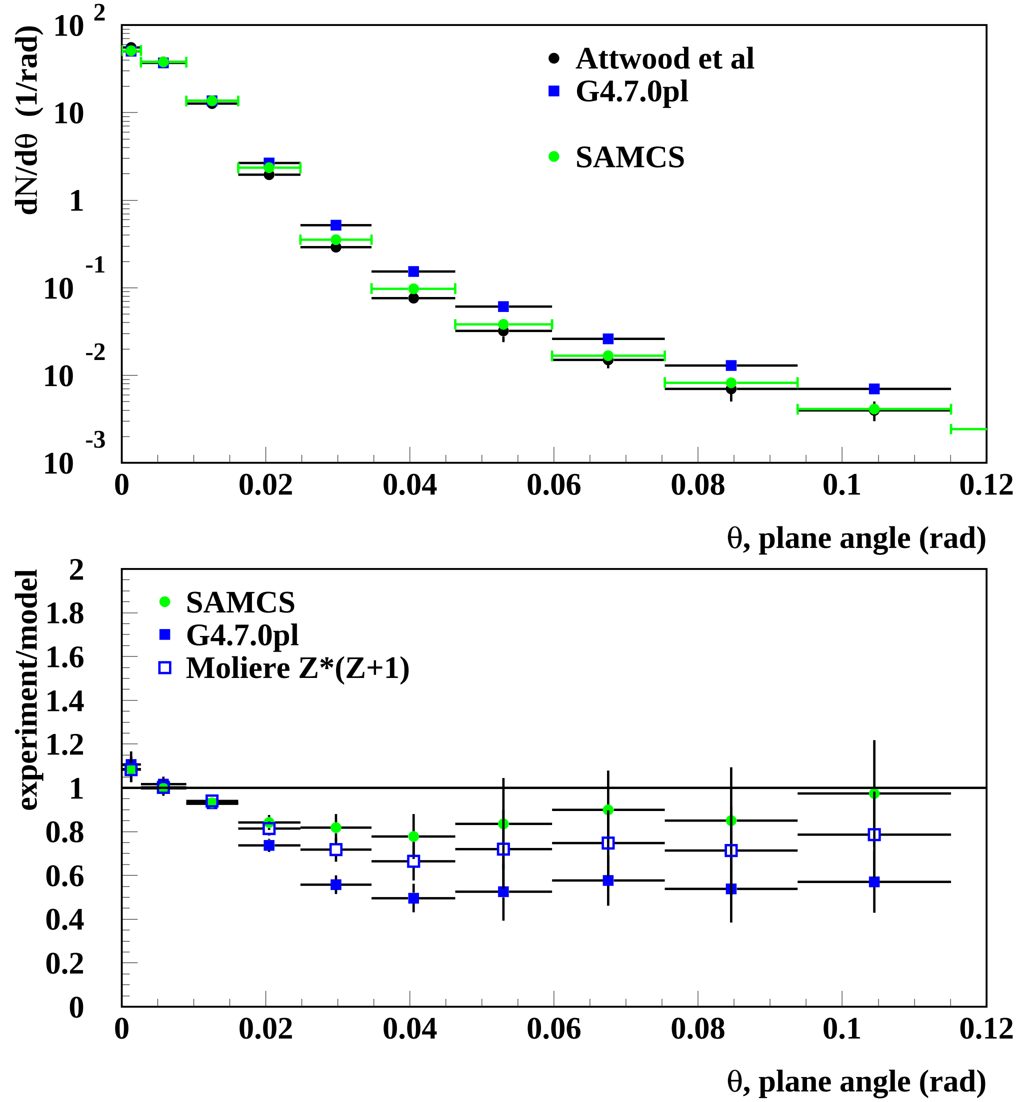
Angular distribution of 172 MeV/c muon after 12.75 mm of lithium

MODELS *vs* MuScat MEASUREMENTS



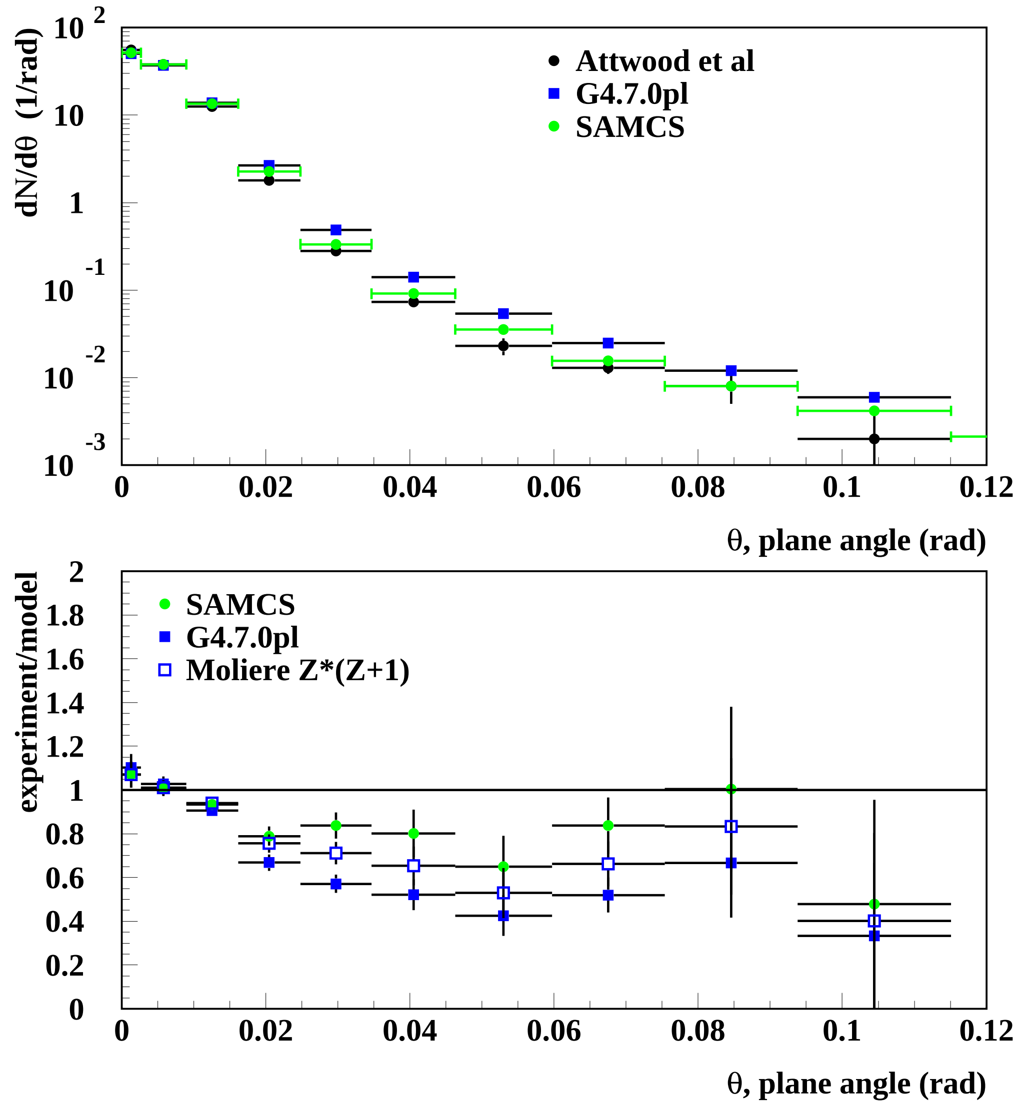
Angular distribution of 172 MeV/c muon after 3.73 mm of beryllium

MODELS *vs* MuScat MEASUREMENTS



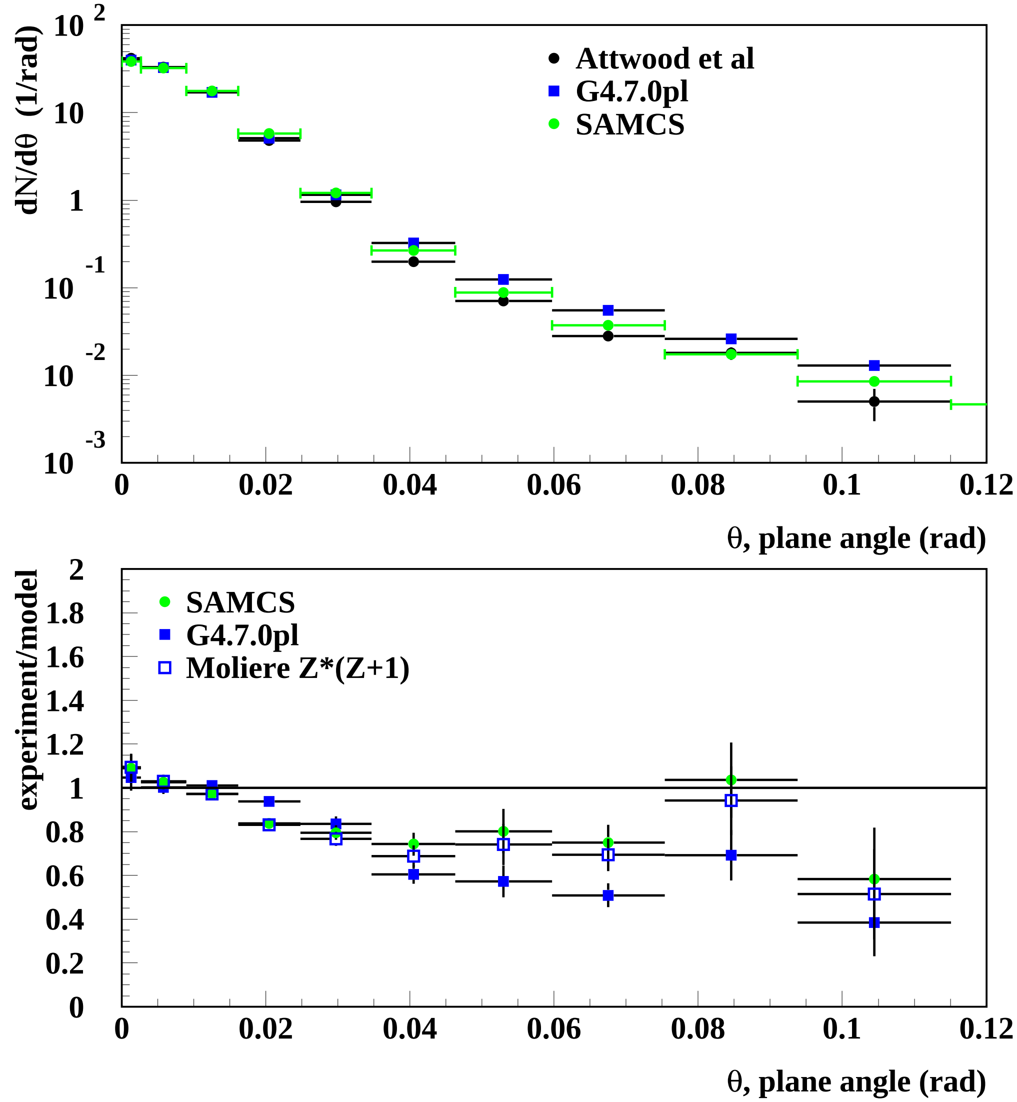
Angular distribution of 172 MeV/c muon after 2.5 mm of carbon

MODELS *vs* MuScat MEASUREMENTS



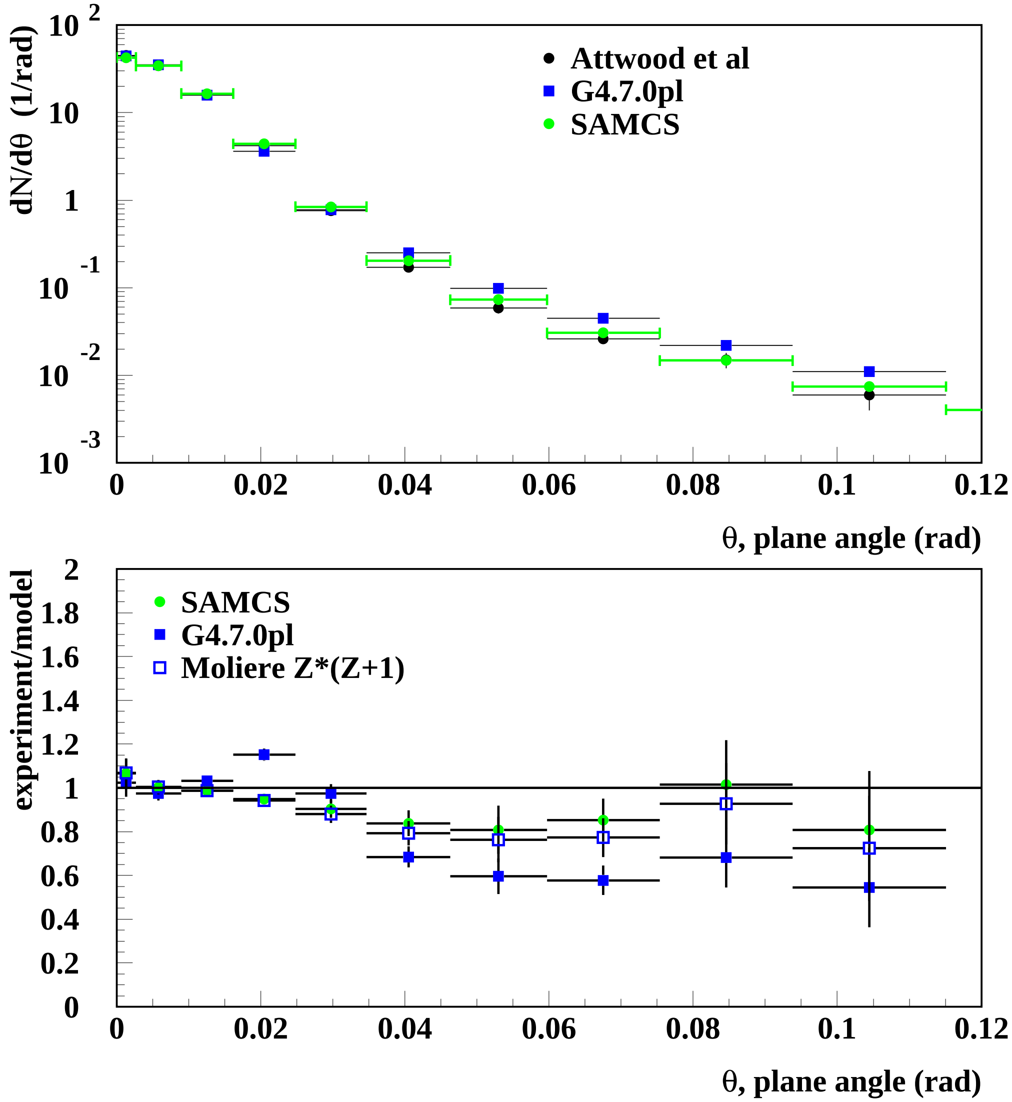
Angular distribution of 172 MeV/c muon after 4.74 mm of CH_2

MODELS *vs* MuScat MEASUREMENTS



Angular distribution of 172 MeV/c muon after 1.5 mm of aluminum

MODELS *vs* MuScat MEASUREMENTS



Angular distribution of 172 MeV/c muon after 0.24 mm of iron

MEAN SQUARED ANGLE

By definition mean squared angle is

$$<\theta^2>= t\int_0^\infty d\Omega\theta^2 \frac{d\Sigma}{d\Omega}$$

Formula Rossi

$$<\theta^2>= (\frac{E_s}{p\beta})^2 \frac{x}{x_0}$$

where $E_s = 21.2$ MeV, p, β are momentum and velocity of projectile, x is scatterer length and x_0 is radiation length.

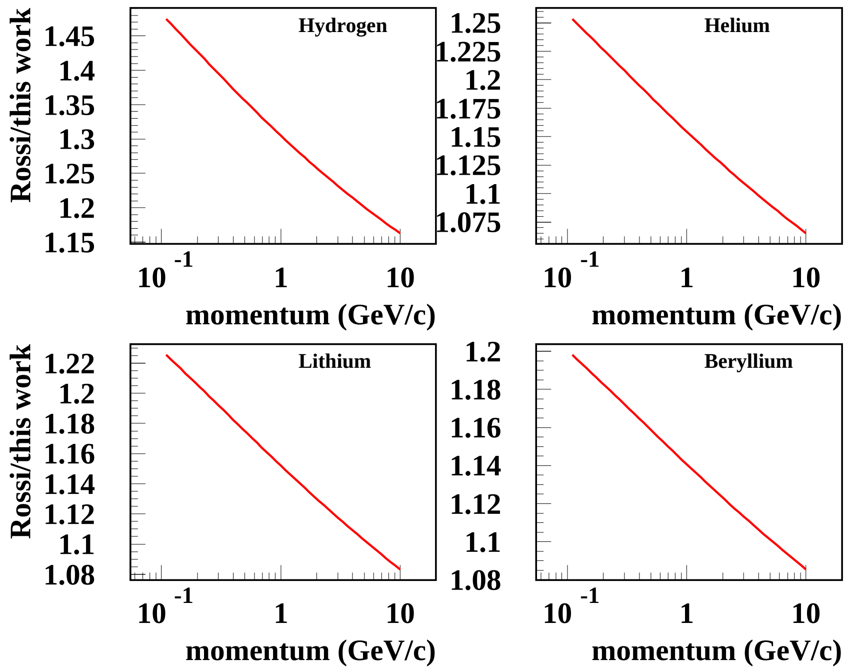
Modified Moliere-Fano theory provides more complicated expression, but for $E_\mu < 1$ GeV it reads

$$<\theta^2>=<\theta_c^2> + <\theta_i^2>,$$

$$<\theta_c^2>= \chi_c^2(ln\frac{r_a^2}{\chi_a^2} - C - 1),$$

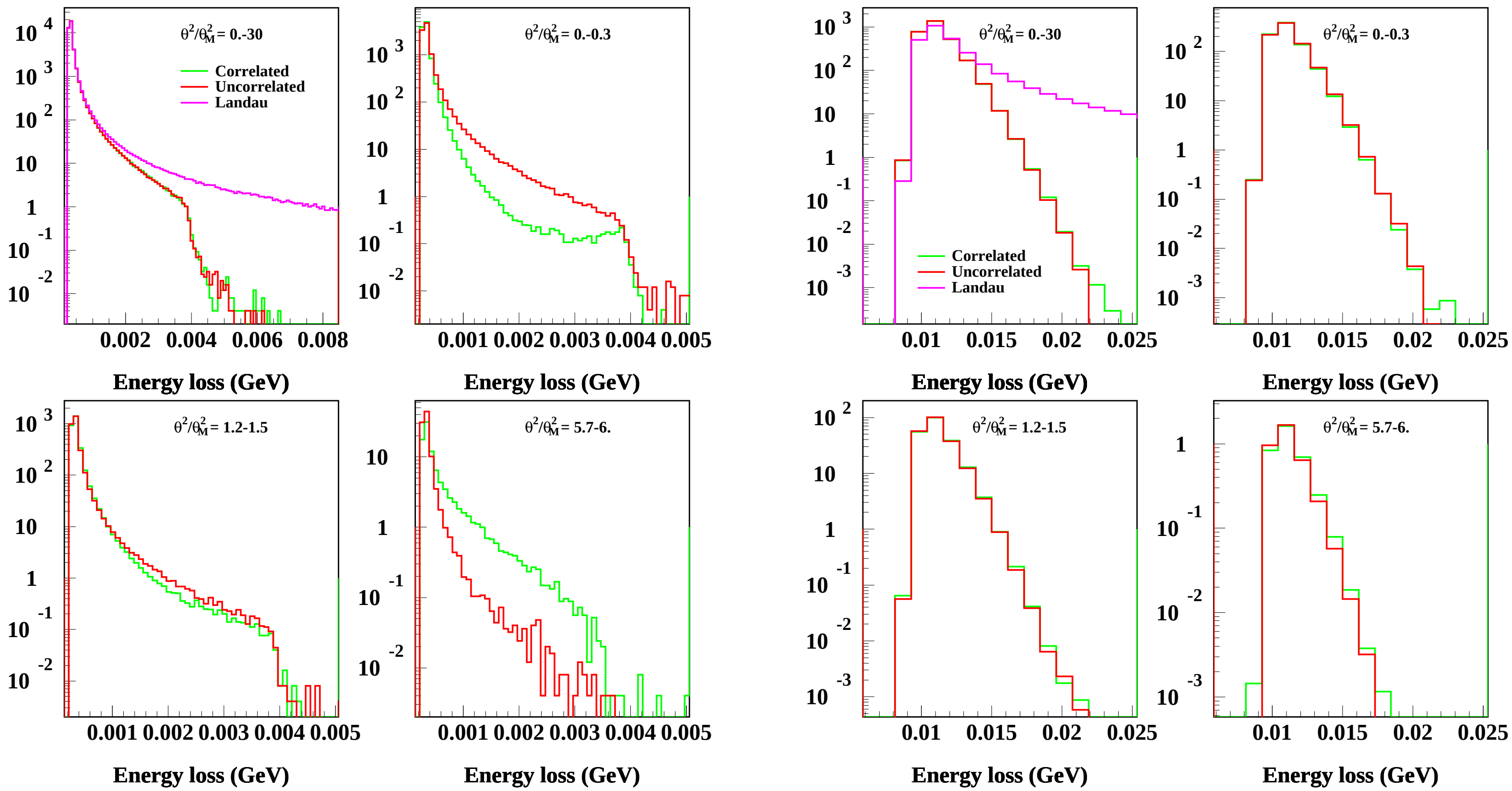
$$<\theta_i^2>= \frac{\chi_c^2}{Z}(ln\frac{2m_eE_{max}}{p^2\chi_i^2} - 2 - \beta^2/2),$$

MEAN SQUARED ANGLE (2)



Muon mean squared angle due to multiple Coulomb scattering

CORRELATED ENERGY LOSS STRAGGLING AND SCATTERING



Energy loss distribution of 200 MeV/c muon

***Left*:** 1 cm liquid hydrogen

***Right*:** 30 cm liquid hydrogen

EFFECT ON MUON COOLING SIMULATIONS

- New generation of models and Monte Carlo codes for simulation of multiple Coulomb scattering are in reasonable agreement with old and new experimental data.
- Six-dimensional emittance is proportional to $< \theta^2_{MCS} >^2$. New models predict much low value of mean squared angle than usually applied.
- Recent ICOOL simulation (Fernov, April 2006) showed big effects of new scattering models on performance of ideal RFOFO cooling ring. For Fano and Tollestrup models the improvement is about 66%, ELMS gives about 150%.
- Energy-angle correlations are not small for thin hydrogen absorber.
- GEANT3/ICOOL “condensed” algorithm for simulation energy loss predicts much wider distribution than SAMCS and Vavilov theory for thin hydrogen absorber.